US waves propagation in an elastic medium: 2D Finite Element Modelling

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Introduction - Scope

US propagation interpretation in solid tricky due to the risk of correlated effects of microstructure evolution + effects of geometry

Perhaps FEM could help to understand and decorrelate the influence of different parameters (texture, grain size, …)

- feasibility?

FEM consists in the division of space in many sub-domains to approximate the global response of a larger structure … scale ratio = number of elements

	$\frac{Larger\ structure\ scale}{Sub - domain\ scale} = \frac{mm}{\mu m}$	
1D	$\frac{mm}{\mu m} \sim 10^3$	- yes, interest?
2D	$\frac{mm^2}{\mu m^2} \sim 10^6$	- possible
3D	$\frac{mm^3}{\mu m^3} \sim 10^9$	- Not on our computers!

Keep in mind … influence in 3D

Outline

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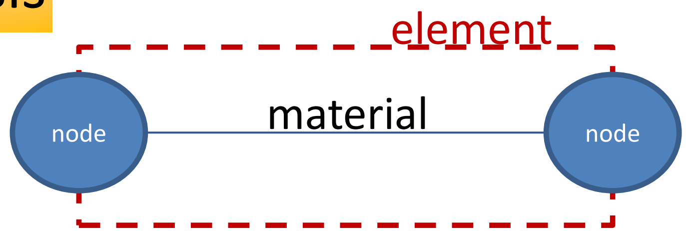
- **Model definition**
  - Explicit dynamic analysis
  - Geometry definition
  - Mesh and Time
  - Materials parameters
- **Model inputs**
  - Pulse (Spatial)
  - Pulse (Temporal)
- **Model outputs**
  - Energy
  - Results
- **Applications and results**
  - Material implementation
  - Example of porosity/inclusions

Conclusion

Model definition

Explicit dynamic analysis

ABAQUS user manual



$\boldsymbol{u}$: position of a node
i : increment number

- Diagonal element mass matrix **M** = mass concentrated at nodes

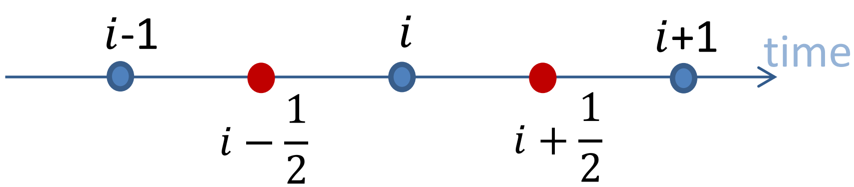
$$\ddot{\boldsymbol{u}}^{(i)} = \boldsymbol{M}^{-1} \cdot \left(\boldsymbol{F}^{(i)} - \boldsymbol{I}^{(i)}\right)$$

$\boldsymbol{F}^{(i)}$: external applied forces
$\boldsymbol{I}^{(i)}$: internal element forces

- Strains and stresses (constitutive laws)

- Explicit integration rule

$$f(i+1) = f(i) + f'(i).\Delta t$$



central difference

$$\begin{cases} \dot{\boldsymbol{u}}^{\left(i+\frac{1}{2}\right)} = \dot{\boldsymbol{u}}^{\left(i-\frac{1}{2}\right)} + \dfrac{\Delta t^{(i+1)} + \Delta t^{(i)}}{2} . \ddot{\boldsymbol{u}}^{(i)} \\ \boldsymbol{u}^{(i+1)} = \boldsymbol{u}^{(i)} + \Delta t . \dot{\boldsymbol{u}}^{\left(i+\frac{1}{2}\right)} \end{cases}$$

Explicit integration rule conditionally stable: above limit of time increment

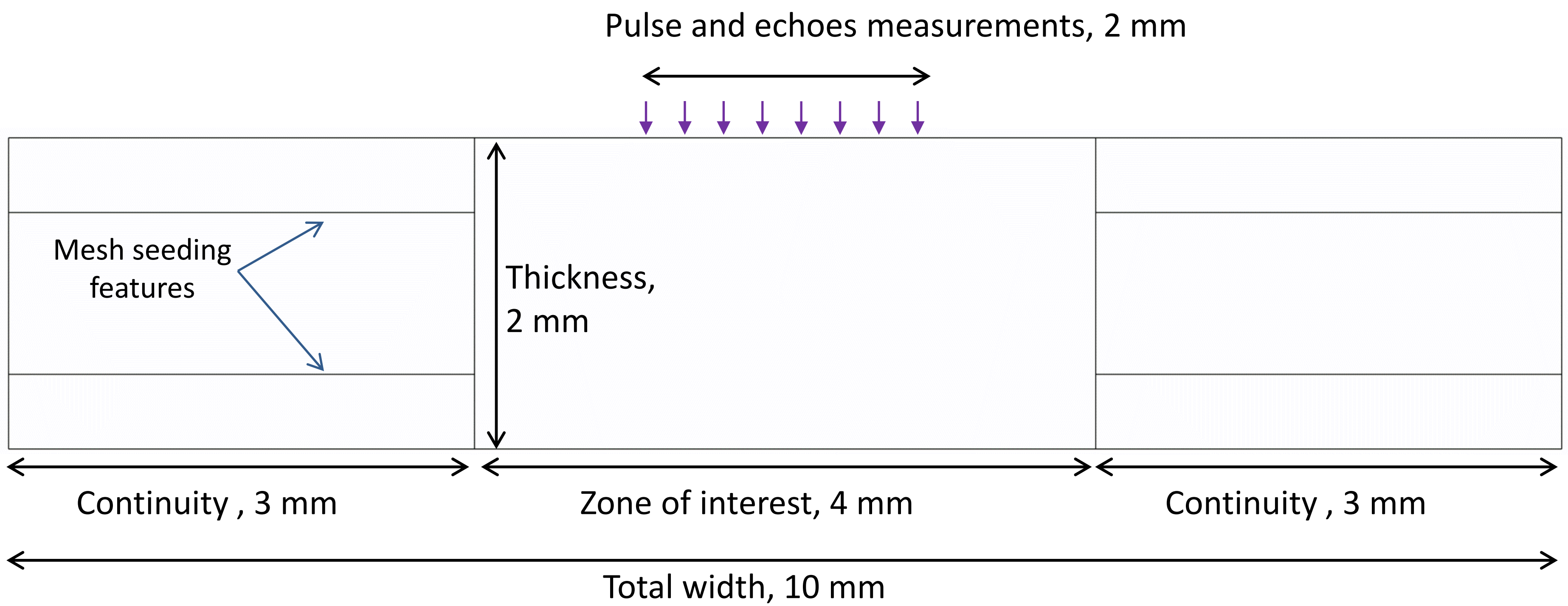
$$\Delta t = min\left(\frac{L_e}{c_d}\right)$$

L_e : element dimension
c_d : dilatational wave speed

Model definition

Geometry

Typical sizes from experimental setup



Model definition

Mesh and time

Element type:

CPE4R

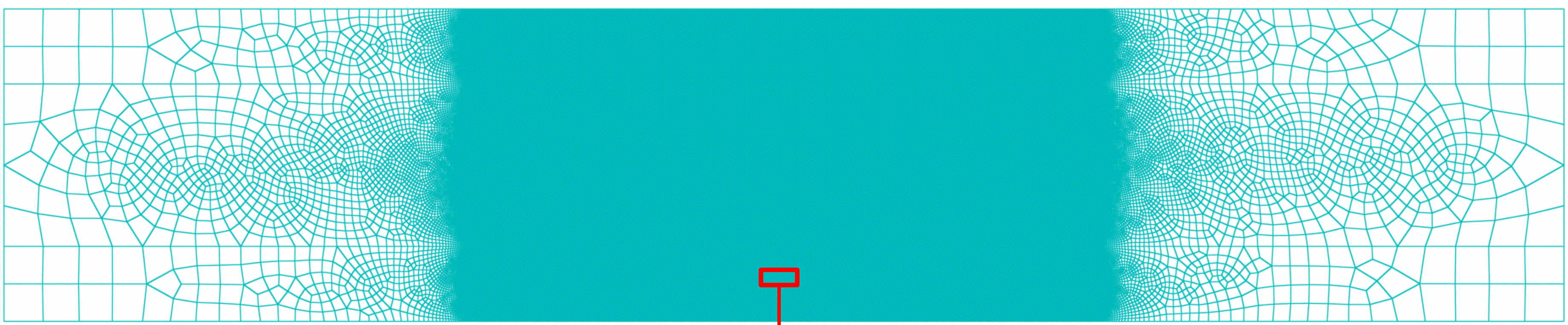
- continuous
- plane strain
- 4 nodes
- reduced integration

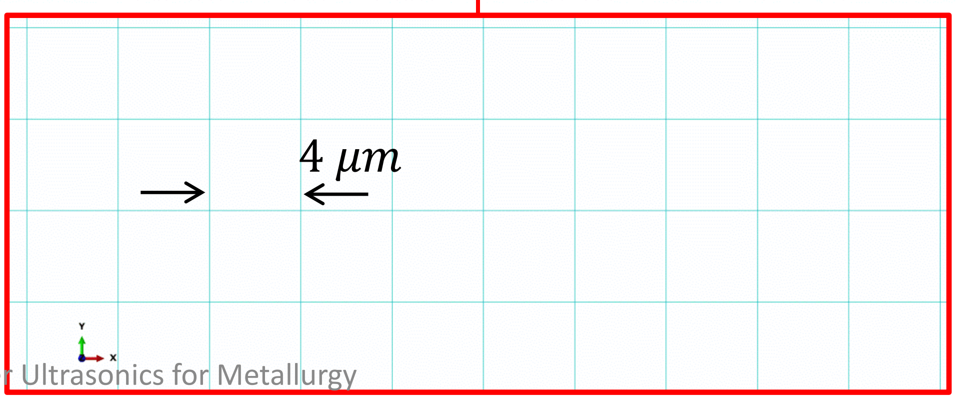
Mesh size ?

There are two spatial scales that have to be consider:

- the minimal wavelength;
- the scale of microstructure (grain size, porosity, inclusion …).

$$\lambda_{min} = \frac{c}{f_{max}} \sim \frac{5000\ m.s^{-1}}{100\ MHz} = 50\ \mu m \Rightarrow L_{min} = \frac{\lambda_{min}}{10} = 5 \mu m$$





$$\Delta t = min\left(\frac{L_e}{c_d}\right) = 8.10^{-10} s$$

$$\Delta t = 2.10^{-10}\ s$$

Model definition

Material parameters

Remark on units: mm - N - tonne - MPa - s - mJ - tonne/mm³

Minimum requested for an isotropic material: ρ, E, υ

For a fully anisotropic material definition: 21 coefficients

$$\begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{12} \\ \sigma_{13} \\ \sigma_{23} \end{Bmatrix} = \begin{bmatrix} D_{1111} & D_{1122} & D_{1133} & D_{1112} & D_{1113} & D_{1123} \\ & D_{2222} & D_{2233} & D_{2212} & D_{2213} & D_{2223} \\ & & D_{3333} & D_{3312} & D_{3313} & D_{3323} \\ & & & D_{1212} & D_{1213} & D_{1223} \\ & sym & & & D_{1313} & D_{1323} \\ & & & & & D_{2323} \end{bmatrix} \begin{Bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ \gamma_{12} \\ \gamma_{13} \\ \gamma_{23} \end{Bmatrix} = [D^{el}] \begin{Bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ \gamma_{12} \\ \gamma_{13} \\ \gamma_{23} \end{Bmatrix}.$$

Material orientation has to be defined, but without out of plane rotations, due to plane strain definition

The rotation are applied to the matrix and implemented only afterwards in the model

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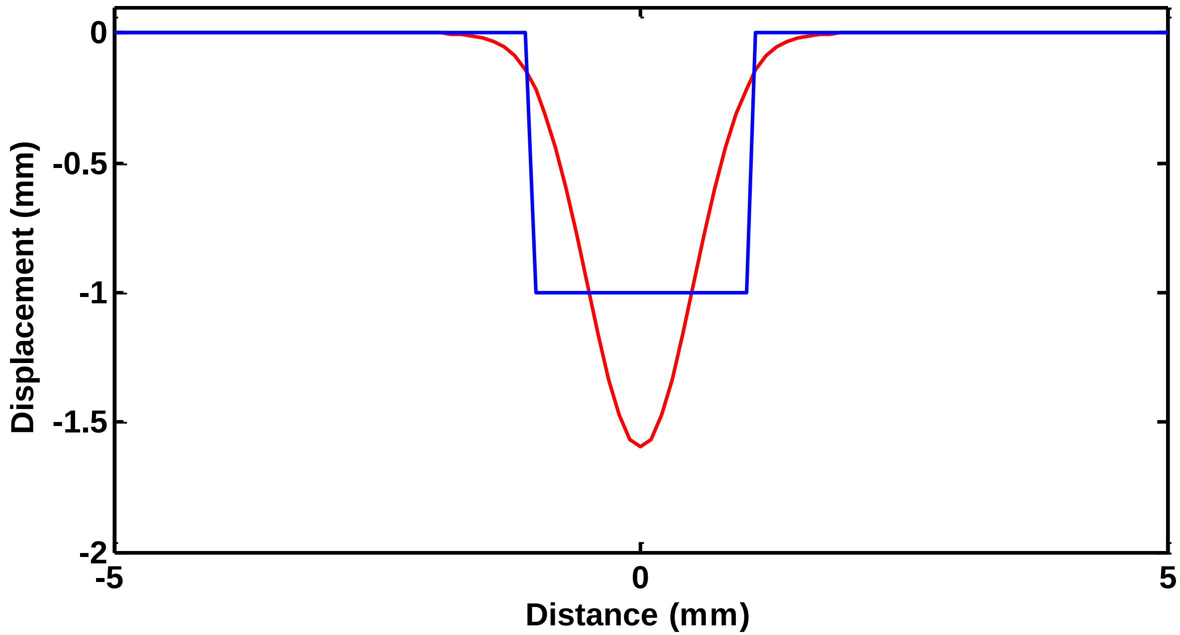
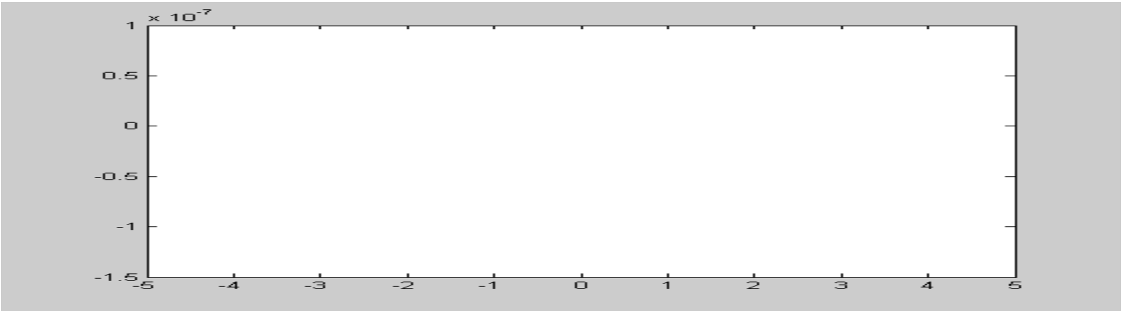
Model input

Pulse spatial definition

Top surface imposed displacement amplitude ≈ 0.1 nm

Shape:

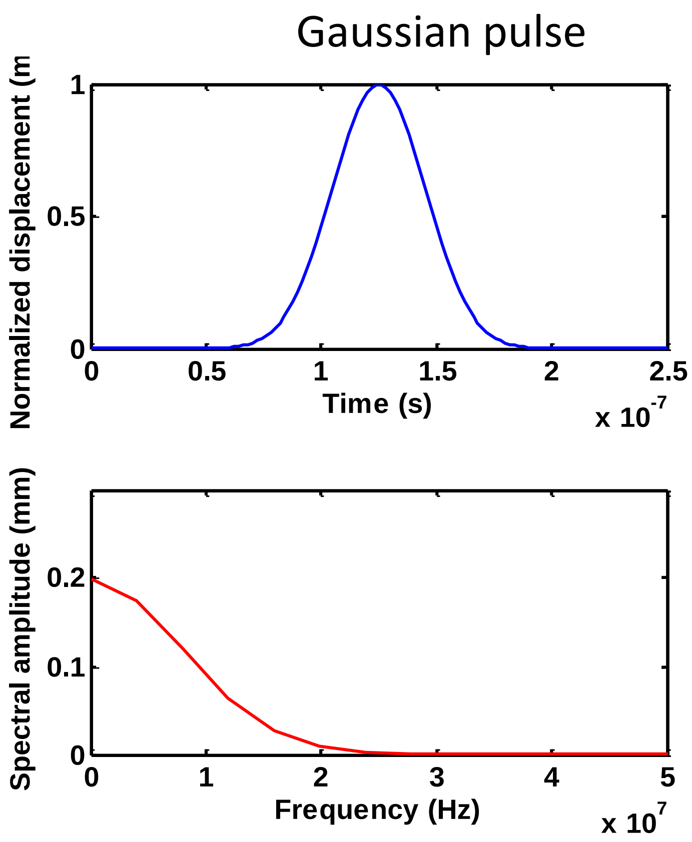




Gaussian pulse is chosen to avoid strong discontinuities

Model input

Pulse temporal definition



Poor frequency content, no control on central frequency.

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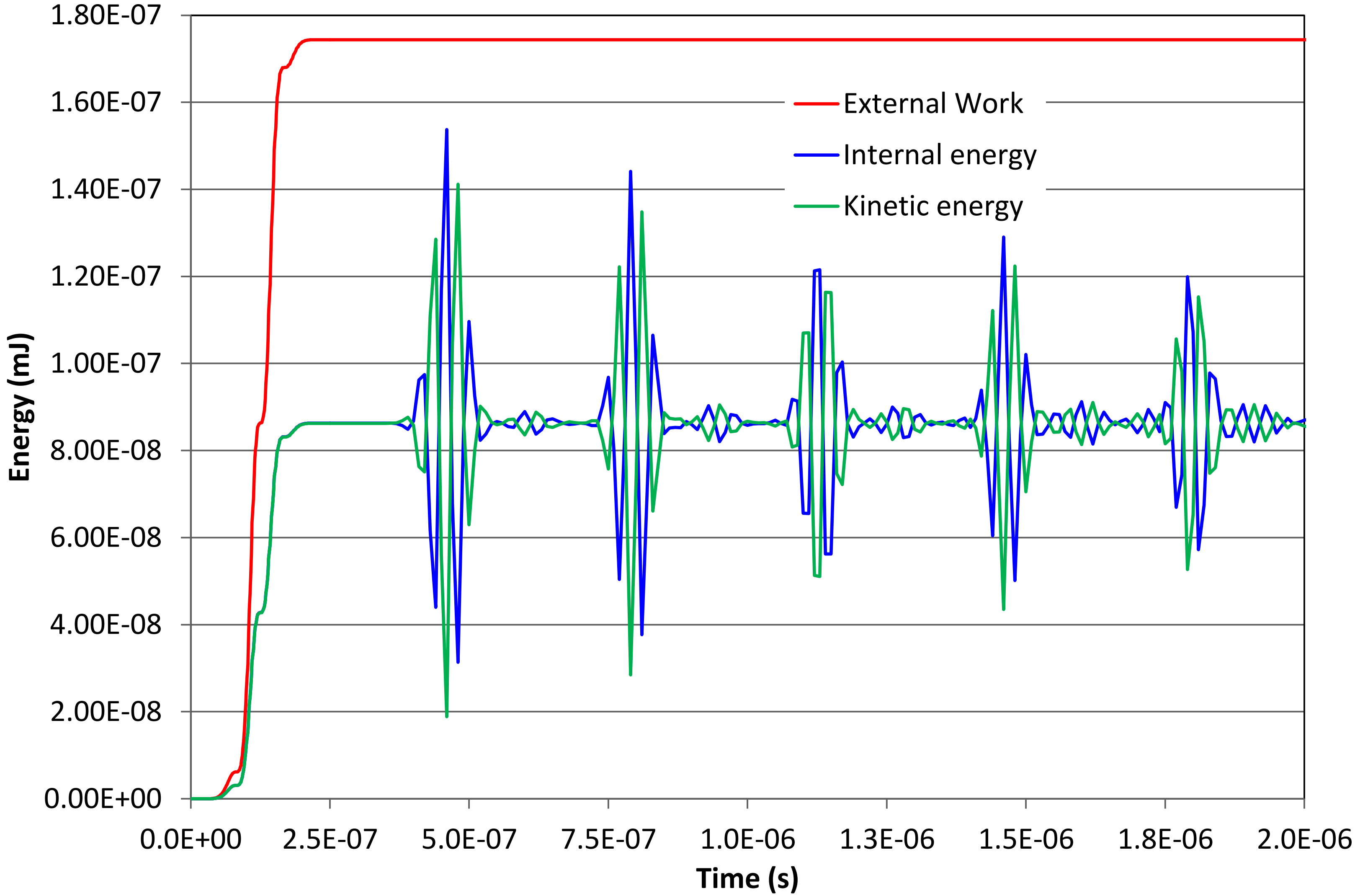
Model outputs

Energy

Examples with isotropic material

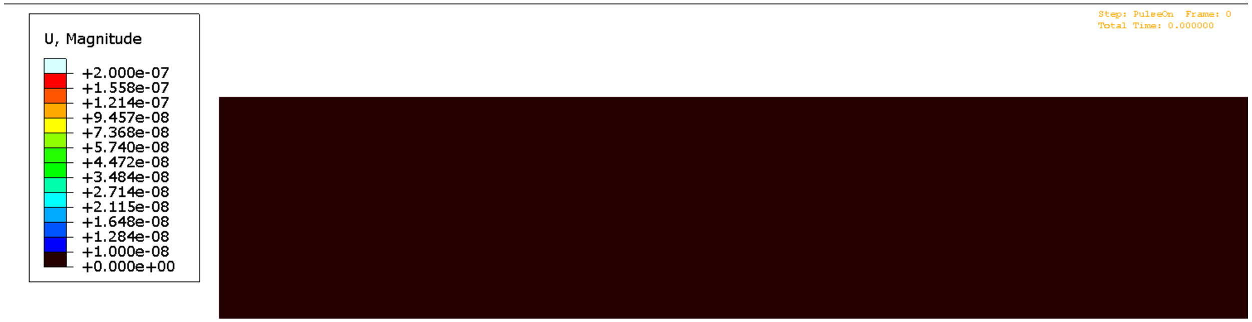


Model outputs

Results

Examples with isotropic material

<u>Contour plot</u>



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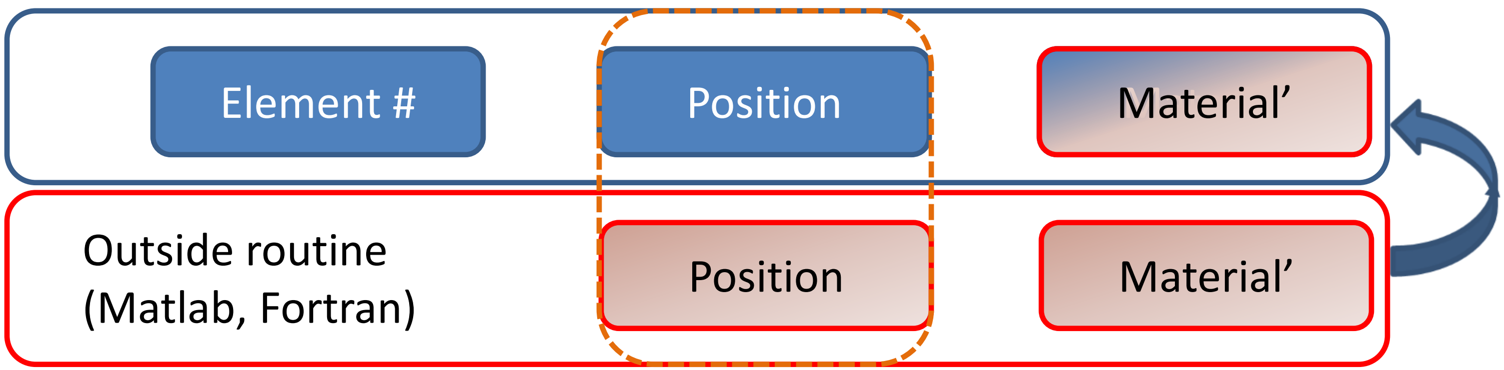
Material implementation

Creation of an input file as template

The inp text file is modified to add microstructural characteristics



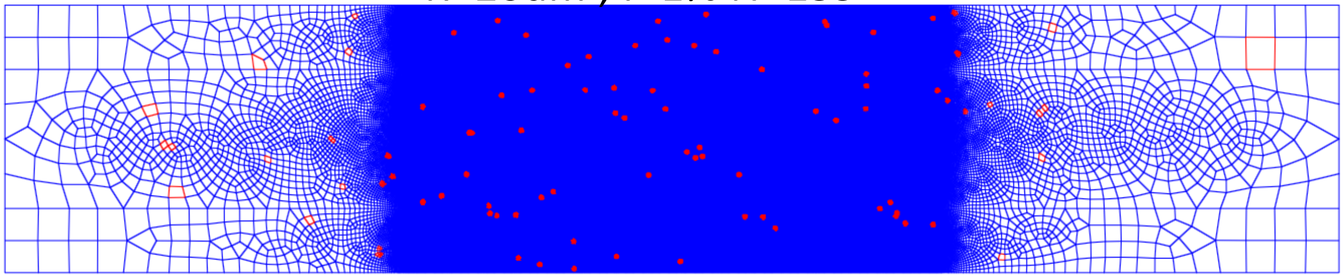
Orientation / type / position of material is defined by an outside routine, and the inp file is appropriately modified

Applications and results

Example of porosity

Ex of routine: *choose randomly a set of coordinate for spherical voids*

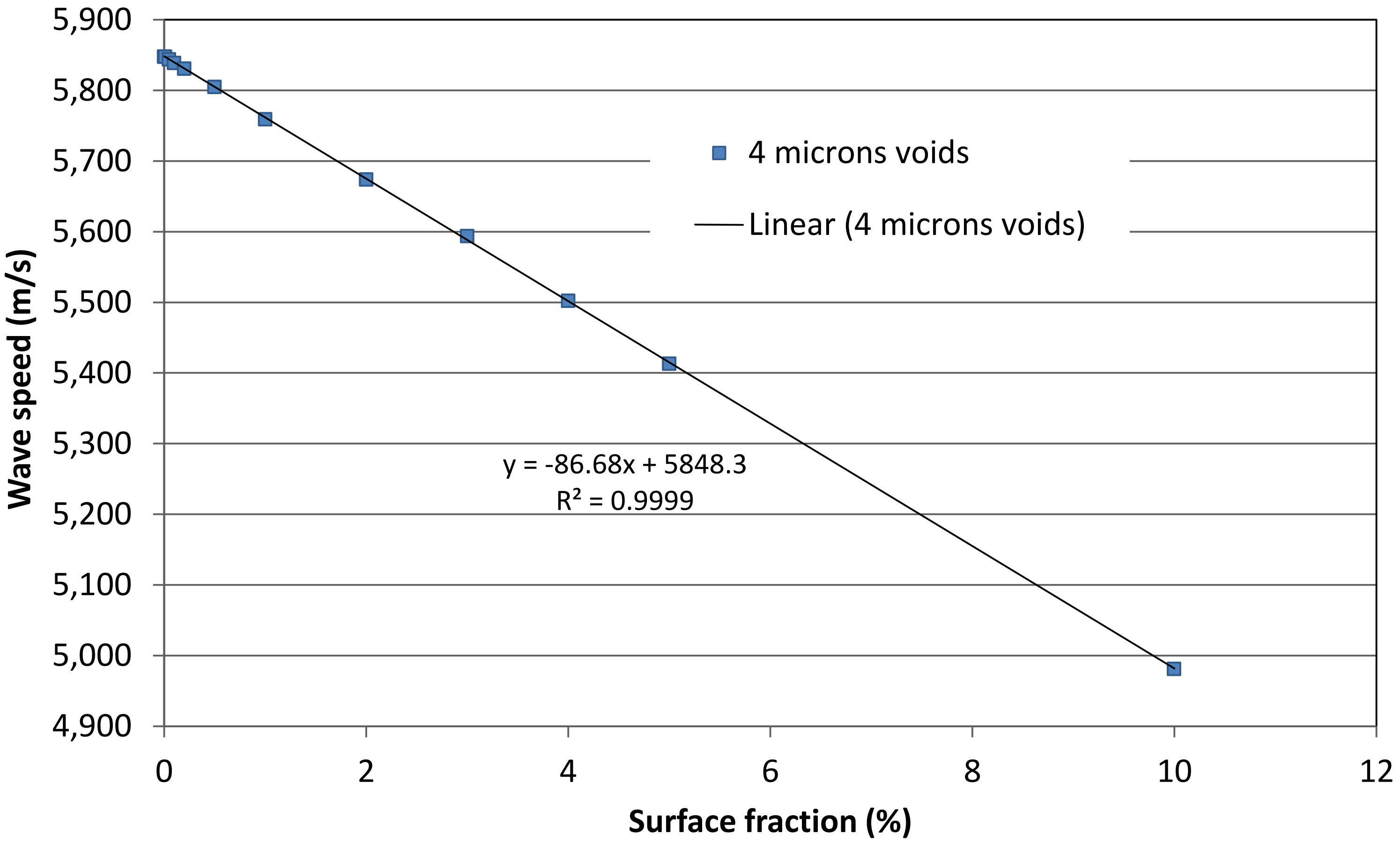
R=20um ; f=1% N=159



Applications and results

Example of porosity

Wave speed evolution for randomly distributed voids of element size

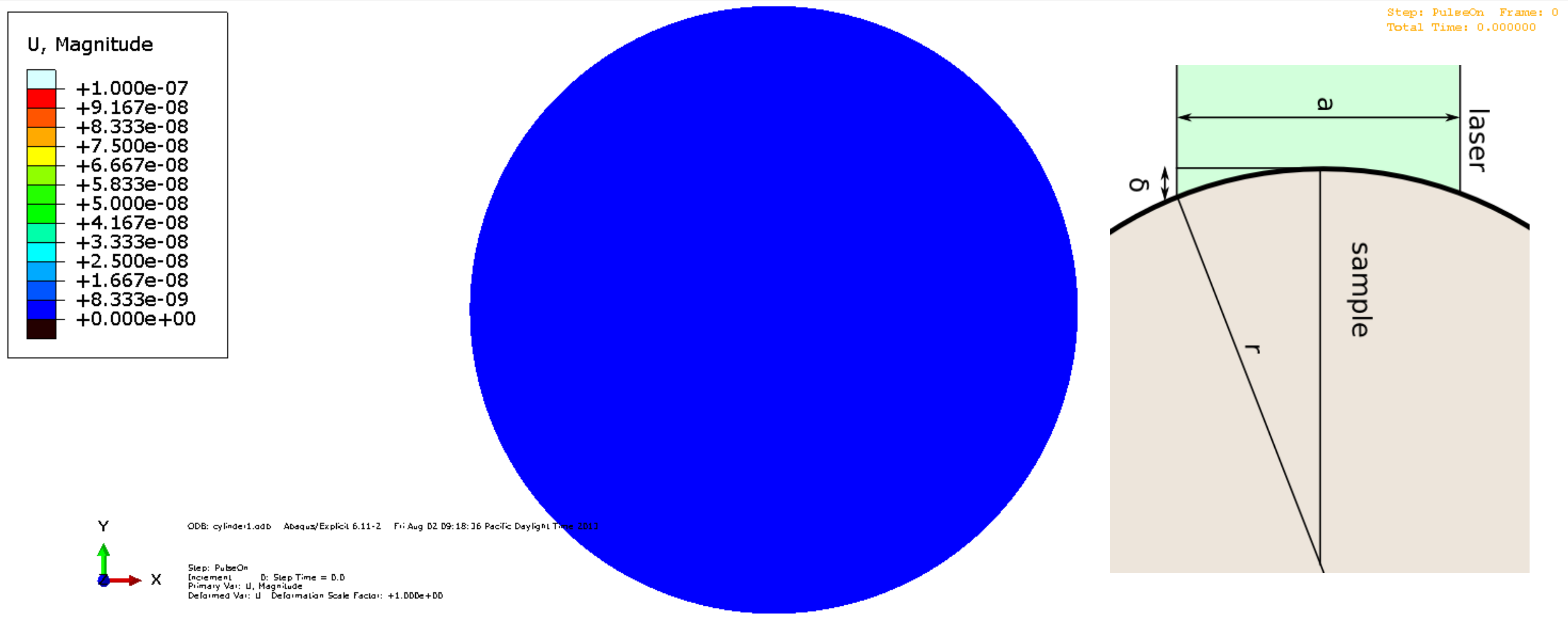


Influence of size? Distribution? Shape?

Applications and results

Effect of geometry: cylinder

Same pulse, apply on top of cylinder.

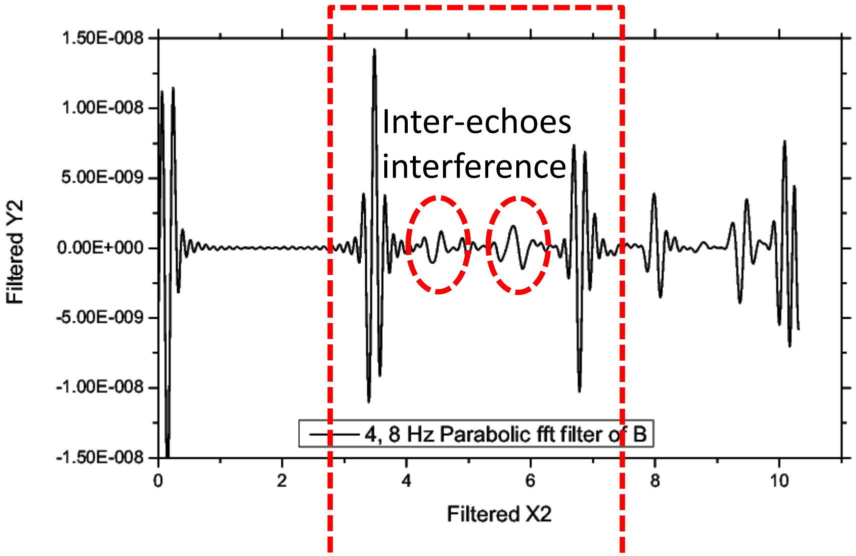


Applications and results

Effect of geometry: cylinder

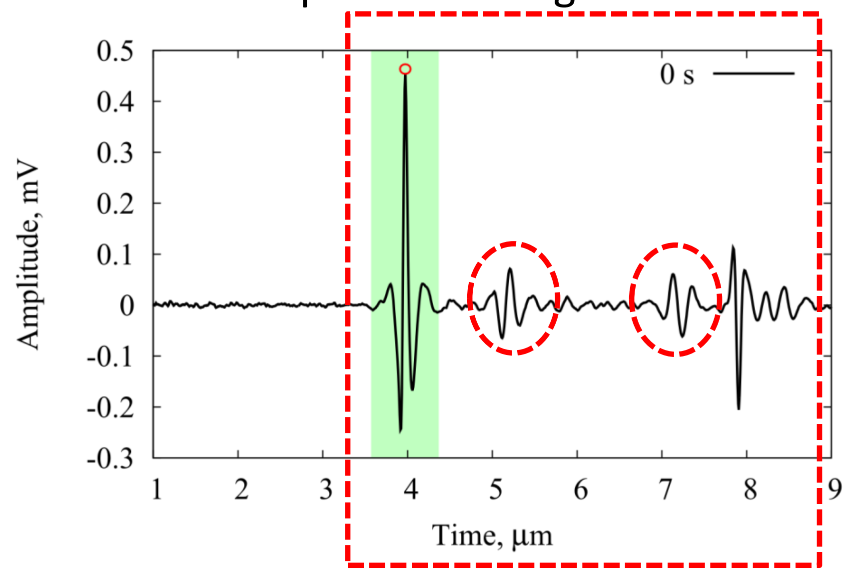
Mean displacement on top surface:

Simulated signal



Inversion of pulse

Experimental signal



The model helps to identify geometrical artefacts

More developed application

Simulation of ultrasound propagation in anisotropic polycrystalline media

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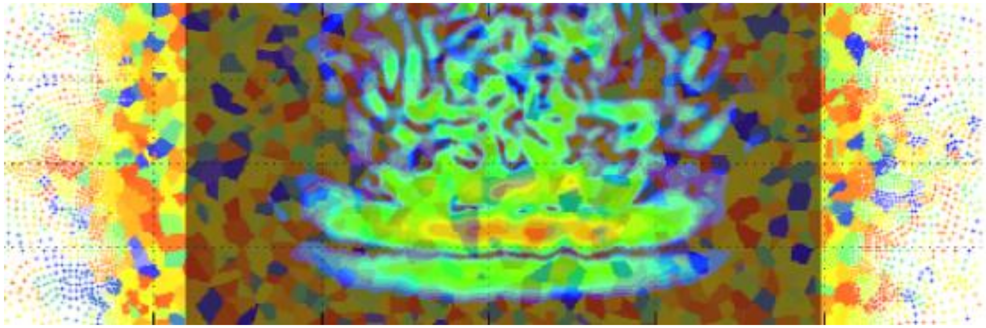
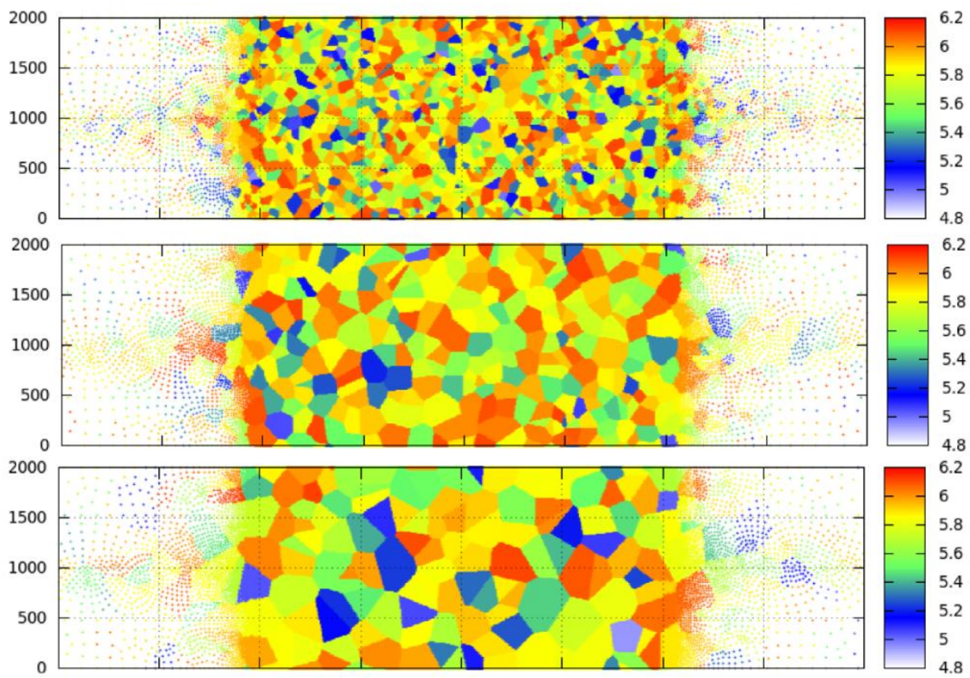
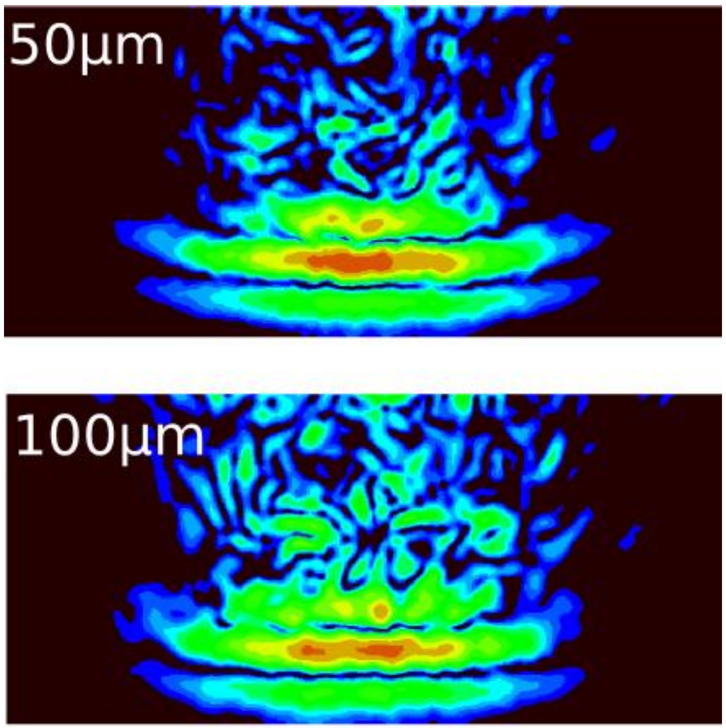


Polycrystalline material

- ✓ Voronoi tessellation
- ✓ Generation of grain size from 20 to 300 μm with random orientation (BCC-iron)

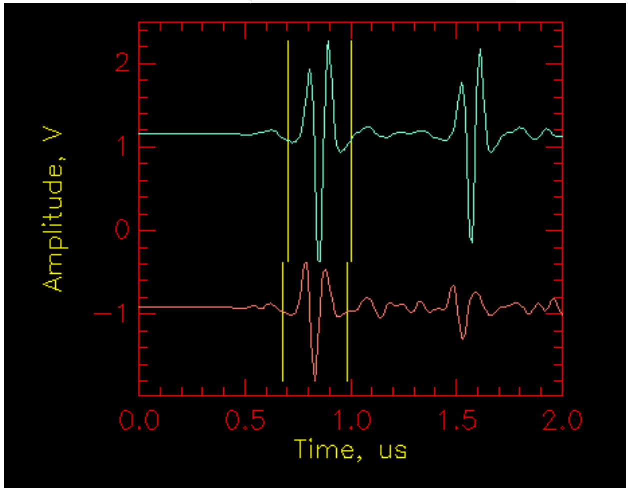




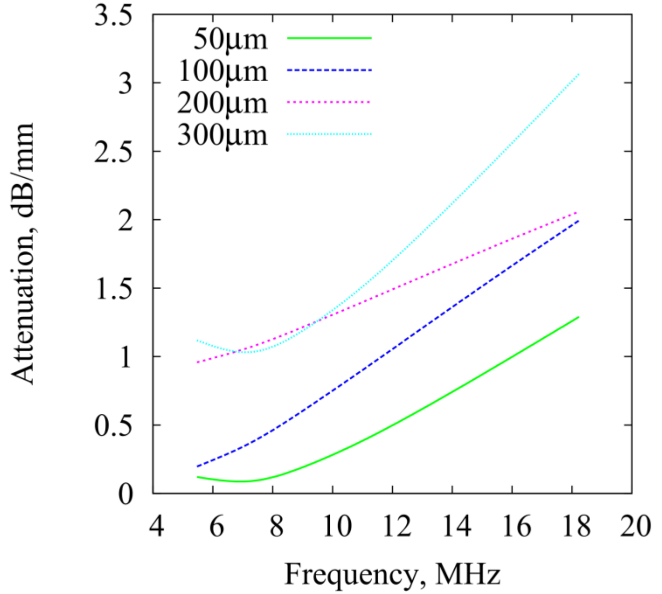
Comparing first echo, Reference 20μm

- ✓ one echoes
- ✓ two waveforms
- ✓ Larger attenuation for larger grain size
- ✓ Examine the frequency dependence

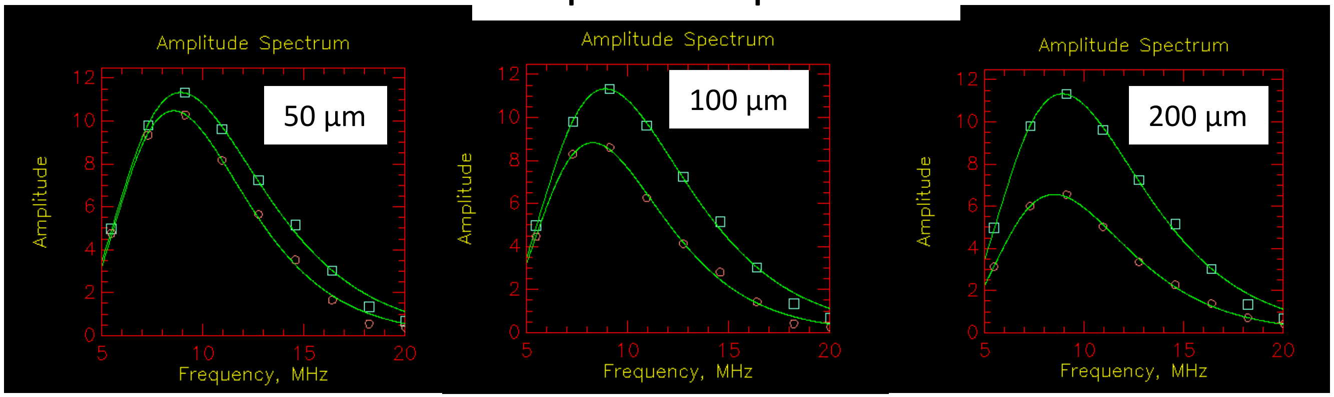
Waveform



Attenuation spectrum



Amplitude spectrum



Conclusions

✓ Three main requisites

Open resource / needs?

✓ Materials properties (stiffness, density, grain size, morphology and crystallographic orientation)

FEM tool

✓ Finite Element engine (Sample geometry, meshing, wave generation, propagation and detection of pulse)

LUMet tool

✓ Waveform analysing tool (Extract ultrasound pulse properties, velocities, attenuation spectrum)

Thank you